arXiv:1007.4921v1 [hep-ex] 28 Jul 2010

On the Sensitivity of L/E Analysis of Super-Kamiokande Atmospheric Neutrino Data to Neutrino Oscillation Part 2

Four Possible L/E Analyses for the Maximum Oscillation by the Numerical Computer Experiment

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Abstract. In the previous paper (Part 1), we have verified that *the SK assumption on the direction* does not hold in the analysis of neutrino events occurred inside the SK detector. Based on the correlation between L_ν and L_μ (Figures 12 and 13 in Part 1) and the correlation between E_ν and E_μ (Figure 14 in Part 1), we have made four possible L/E analyses, namely L_ν/E_ν, L_ν/E_μ, L_μ/E_ν and L_μ/E_μ. Among four kinds of L/E analyses, we have shown that only L_ν/E_ν analysis can give the signature of maximum oscillations clearly, not only the first maximum oscillation but also the second and third maximum oscillation, while the L_μ/E_μ analysis which are really done by Super-Kamiokande Collaboration cannot give the maximum oscillation at all. It is thus concluded from those results that the experiments with the use of the cosmic-ray beam for neutrino oscillation, such as Super-Kamiokande type experiment, cannot find the maximum oscillation from L/E analysis, because the incident neutrino cannot be observed due to its neutrality. Therefore, we would suggest Super-Kamiokande Collaboration to re-analyze the zenith angle distribution of the neutrino events which occur inside the detector carefully, because L_ν and L_μ are alternative expressions of the cosine of the zenith angle for the incident neutrino and that for the emitted muon, respectively.

PACS. 1 3.15.+g, 14.60.-z

1 Introduction

In Figures 12 and 13 of the preceding paper[1], we have shown that *the SK assumption on the direction* that the directions of the incident neutrinos are the same as those of the emitted muons does not hold. Also, in Figure 14 of the same paper, we have shown that the energies of the incident neutrinos cannot be determined from the those of the emitted muons, uniquely. However, the discrepancies between two variables in Figures 12 and 13 are distinctively large compared with those in Figure 14. Therefore, non-holding of *the SK assumption on the direction* plays an essential role in the L/E analysis for finding the maximum oscillation (oscillation pattern in neutrino oscillation).

The survival probability of a given flavor is given in Eq.(1), in the case of Super-Kamiokande Collaboration. The variables for the L/E analysis are L_ν and E_ν, where L_ν denotes the flight length for the incident neutrino between the generation point of the incident neutrino and the interaction point of the neutrino concerned in the detector, and E_ν is the energy of the incident neutrino.

2 L/E Distributions in Our Numerical Computer Experiment

Our computer numerical experiments are carried out in the unit of 1489.2 days. Hereafter, we call 1489.2 live days as one SK live day. The live days of 1489.2 is the total live days for the analysis of the neutrino events generated inside the detector by Super-Kamiokande Collaboration [2]. We repeat one SK live day experiment as much as 25 times, namely, the total live days for our computer numerical experiments is 37230 live days (25 SK live days). In Figure 1, we show L_ν/E_ν distribution without oscillation for one experiment (1489.2 live days) among twenty five computer numerical experiments. In those numerical

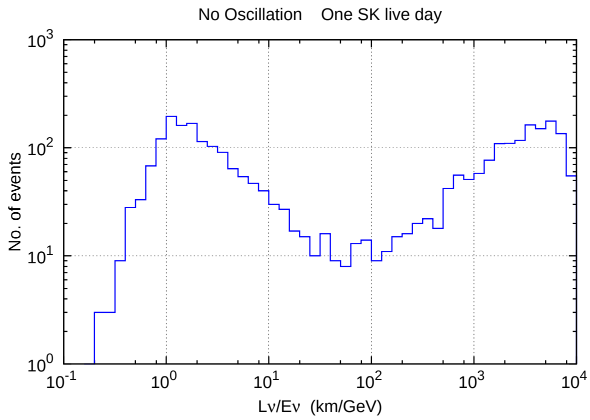

Fig. 1. L_ν/E_ν distribution without oscillation for 1489.2 live days (one SK live day).

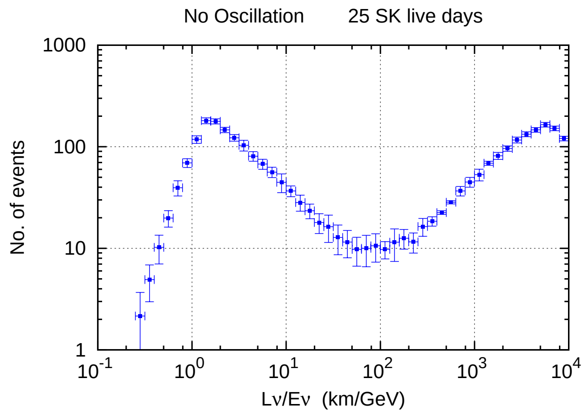

Fig. 2. L_ν/E_ν distribution without oscillation for 37230 live days (25 SK live days).

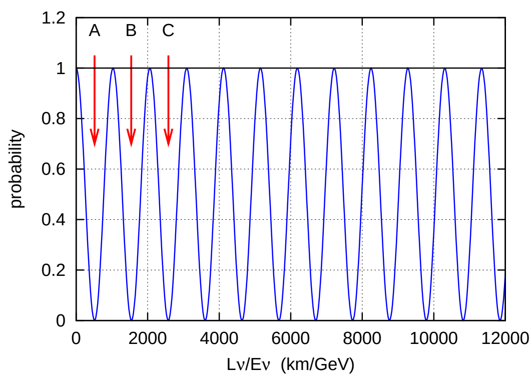

Fig. 3. Survival probability of $P(\nu_\mu \to \nu_\mu)$ as a function of L_ν/E_ν under the neutrino oscillation parameters obtained by Super-Kamiokande Collaboration .

experiments, there are statistical uncertainties only which are due to both the stochastic character in the physical processes concerned and the geometry of the detectors. Therefore we add the standard deviation as for the statistical uncertainty around their average in the forthcoming graphs, if neccessary. In Figure 2, we show the statistical uncertainty, the standard deviations around their average values through twenty five experiments. Similarly for other possible combinations of L and E (L_ν/E_μ, L_μ/E_ν and L_μ/E_μ) for 37230 live days (25 SK live days) we did so.

2.1 L_ν/E_ν distribution

2.1.1 For null oscillation

In Figures 1 and 2, both distributions show the sinusoidal-like character for L_ν/E_ν distribution, namely, the appearance of the top and the bottom, even for null oscillation. The uneven histograms in Figure 1, comparing with those in Figure 2, show that the statistics of Figure 1 is not enough compared with that of Figure 2. Roughly speaking, smaller L_ν/E_ν correspond to the contribution from downward neutrinos, larger L_ν/E_ν correspond to that from upward neutrinos and L_ν/E_ν near the minimum correspond to the horizontal neutrinos, although the real situation is more complicated, because the backscattering effect in QEL as well as the azimuthal angle effect in QEL could not be neglected. From Figure 2, we understand that the bottom around 70 km/GeV denotes the contribution from the horizontal direction and has no relation with neutrino oscillation in any sence.

2.1.2 For oscillation (SK oscillation parameters)

The survival probability of a given flavor, such as ν_μ, is given by

$$P(\nu_\mu \to \nu_\mu) = 1 - sin^2 2\theta \cdot sin^2(1.27\Delta m^2 L_\nu/E_\nu). \quad (1)$$

Then, for maximum oscillations under SK neutrino oscillation parameters, we have

$$1.27\Delta m^2 L_\nu/E_\nu = (2n+1)\times\frac{\pi}{2}, \quad (2)$$

where $\Delta m^2 = 2.4\times 10^{-3}\mathrm{eV}^2$. From Eq.(2), we have the following values of L_ν/E_ν for maximum oscillations.

$$\begin{aligned} L_\nu/E_\nu &= 515\mathrm{km/GeV} \quad for\ n=0 \quad (3-1)\\ &= 1540\mathrm{km/GeV} \quad for\ n=1 \quad (3-2)\\ &= 2575\mathrm{km/GeV} \quad for\ n=2 \quad (3-3)\end{aligned}$$

and so on.

In Figure 3, we give the survival probability $P(\nu_\mu \to \nu_\mu)$ as a function of L_ν/E_ν under the neutrino oscillation parameters obtained by Super-Kamiokande Collaboration.

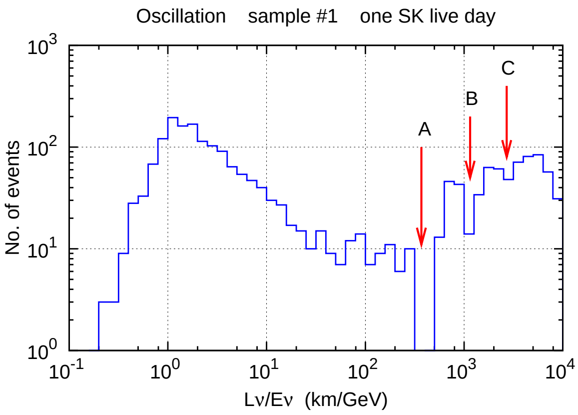

Fig. 4. L_ν/E_ν distribution with oscillation for 1489.2 live days (one SK live day), sample No.1.

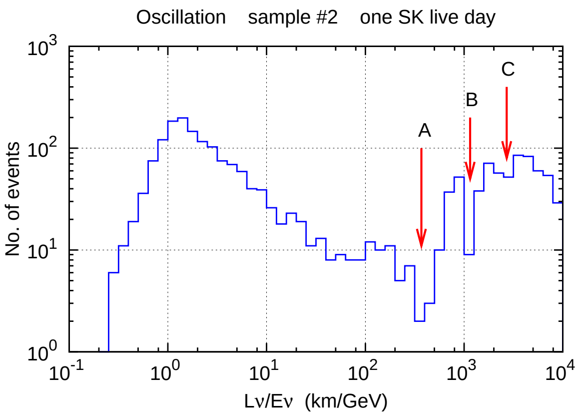

Fig. 5. L_ν/E_ν distribution with oscillation for 1489.2 live days (one SK live day), sample No.2.

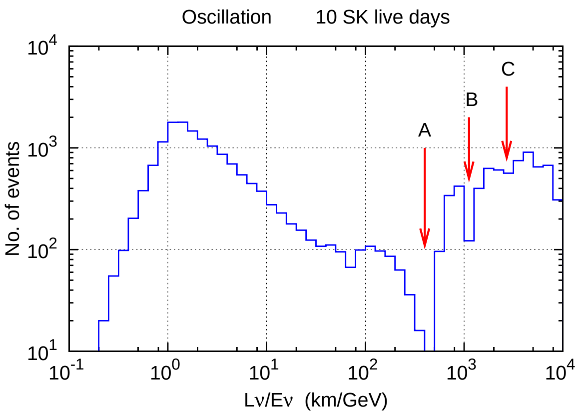

Fig. 6. L_ν/E_ν distribution with oscillation for 14892 live days (10 SK live days).

In cosmic ray experiments, the energy spectrum of the incident neutrino, is convoluted into the survival probability.

In Figure 4, we give one example of the L_ν/E_ν distribution for one SK live day (1489.2 live days)[2] among twenty five sets of the computer numerical experiments in the unit of one SK live day. In Figure 5, we give another example for one SK live day. Arrows A, B and C represent the first, the second and the third maximum oscillation which are given in Eq. (3-1), (3-2) and (3-3), respectively. By the definition of our computer numerical experiments, there are no experimental error bars in L_ν/E_ν distributions in Figures 4 and 5.

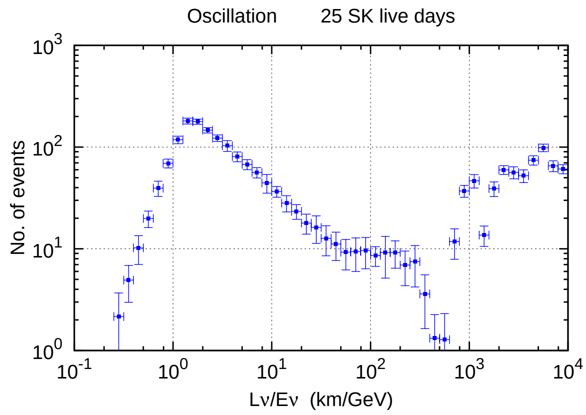

Fig. 7. L_ν/E_ν distribution with standard deviations with oscillation for 37230 live days (25 SK live days).

In Figure 6, we show the L_ν/E_ν distribution for 14892 live days (10 SK live days). Compared Figure 620 with Figures 4 and 5, it is clear that L_ν/E_ν distribution in Figure 6 becomes smoother due to larger statistics. In Figure 7, we can add the statistical uncertainty (standard deviation in this case) around their average, because every one SK live day experiment among twenty five sets of the experiments fluctuates one by one due to their stochastic character in their physical processes and geometrical conditions of the detectors concerned. In order to make the image of the maximum oscillations in L_ν/E_ν distributions clearer, we show the correlations between L_ν and E_ν in Figures 8 and 9, which correspond to Figures 4 and 6, respectively. In Figure 8 for one SK live day, we can observe vacant regions for the events concerned assigned by A, B and C. In Figure 9 for ten SK live days, the existence of the vacant regions for the events concerned becomes clearer due to larger statistics.

In Figure 10, we give L_ν/E_ν distribtution with 14892 live days (10 SK live days) in the linear scale which is another expression of the same content as in Figure 9. Also, it is the survival probability convoluted with the incident neutrino energy spectrum. If we compare Figure 10 with Figure 3, then, we clearly see the series of maximum oscillations characterized with n=0 (A), 1(B), 2(C) and so on which are given by Eq.(2). It is clear from Figure 10 that the maximum oscillations with n=0,1 and 2 have the almost same frequencies [1] under the incident neutrino energy spectrum utilized by Super-Kamiokande Collaboration [3] (see footnote 1). The situation shown in Figures 8 to 10 shows definitely that our computer numerical exper-

[1] Super-Kamiokande Collaboration never mentioned existence of the second and the third maximum oscillations (n=1 and 2)

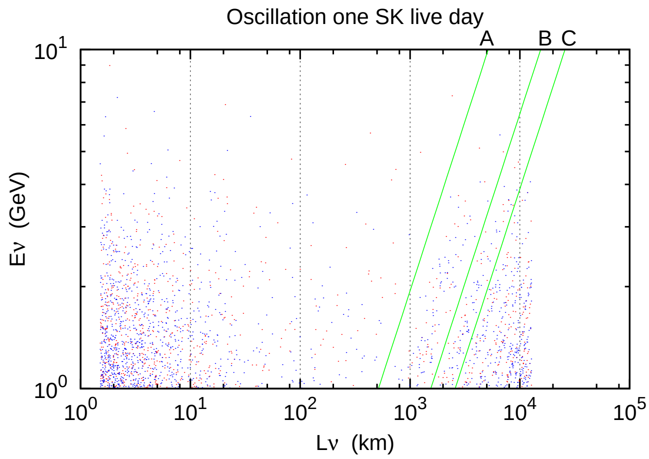

Fig. 8. The correlation diagram between L_ν and E_ν with oscillation for 1489.2 live days (one SK live day).

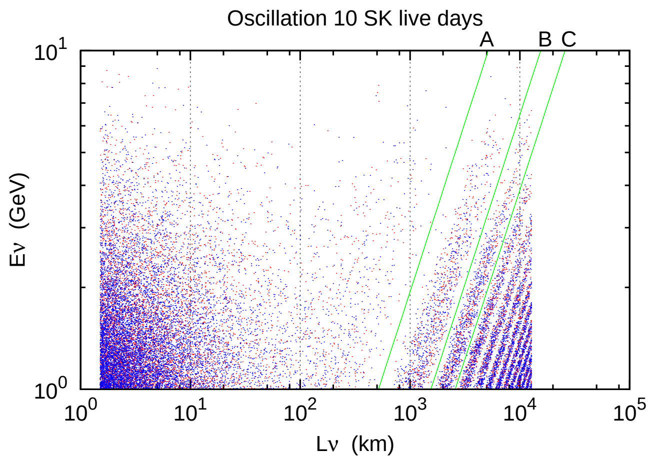

Fig. 9. The correlation diagram between L_ν and E_ν with oscillation for 14892 live days (10 SK live days).

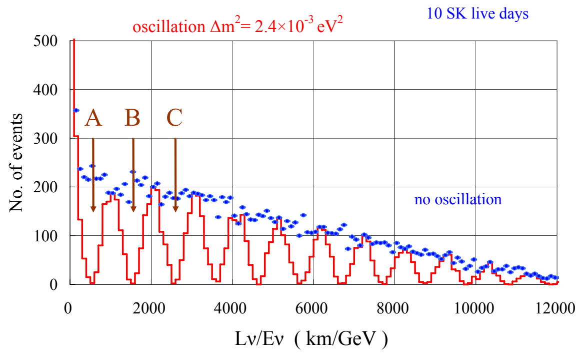

Fig. 10. L_ν/E_ν distribution with and without oscillation for 14892 live days (10 SK live days).

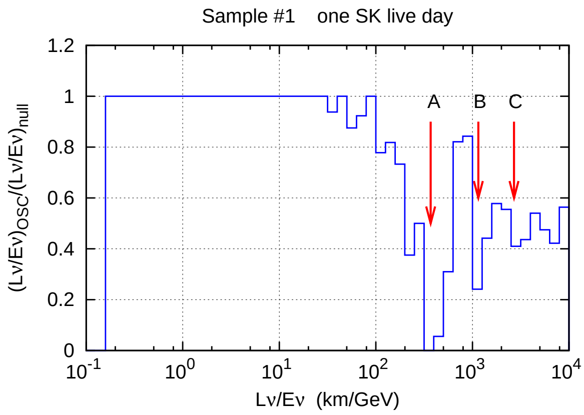

Fig. 11. The ratio of $(L_\nu/E_\nu)_{osc}/(L_\nu/E_\nu)_{null}$ for 1489.2 live days (one SK live day).

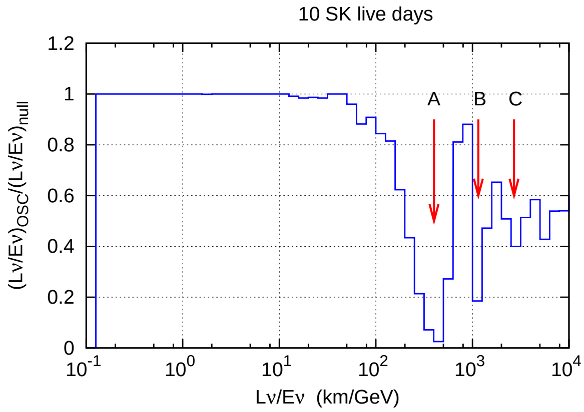

Fig. 12. The ratio of $(L_\nu/E_\nu)_{osc}/(L_\nu/E_\nu)_{null}$ for 14892 live days (10 SK live days).

iment are carried out as exactly as possible from the view point of the stochastic treatment to the matter.

We have repeated the computer numerical experiment for one SK live day as much as twenty five times independently, in both cases with oscillation and without oscillation. Consequently, there are 625 (= 25 × 25) sets of ratios of $(L_\nu/E_\nu)_{osc}/(L_\nu/E_\nu)_{null}$ for one SK live day which correspond to Eq.(1). In Figure 11, we show one example among 625 combinations. In Figure 12, we show the same ratio for 14892 live days (10 SK live days). In conclusion, from Figures 4 to 12, we can reproduce the minimum extrema for neutrino oscillation in our L_ν/E_ν analysis. This fact shows doubtlessly that our computer numerical experiments are done in the correct manner.

2.2 L_μ/E_μ distribution

As physical quantities which can really be observed are L_μ and E_μ instead of L_ν and E_ν, therefore we examine L_μ/E_μ distribution focusing the existence of the maximum oscillation.

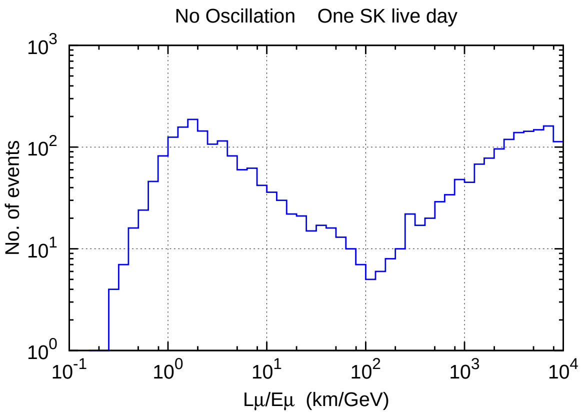

Fig. 13. L_μ/E_μ distribution without oscillation for 1489.2 live days (one SK live day).

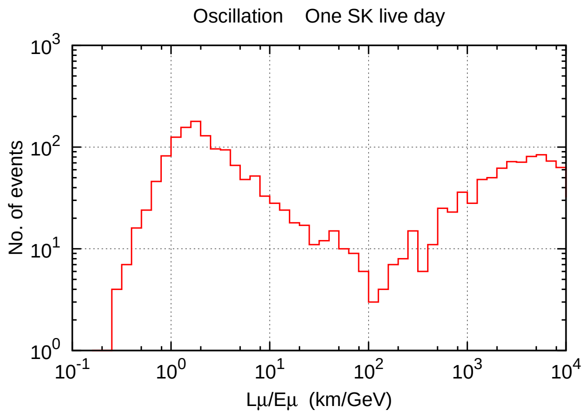

Fig. 15. L_μ/E_μ distribution with oscillation for 1489.2 live days (one SK live day).

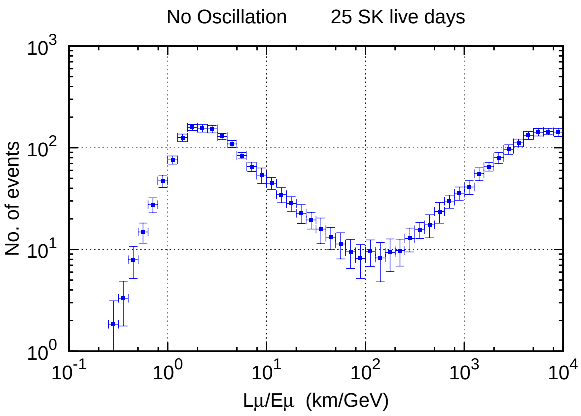

Fig. 14. L_μ/E_μ distribution without oscillation for 37230 live days (25 SK live days).

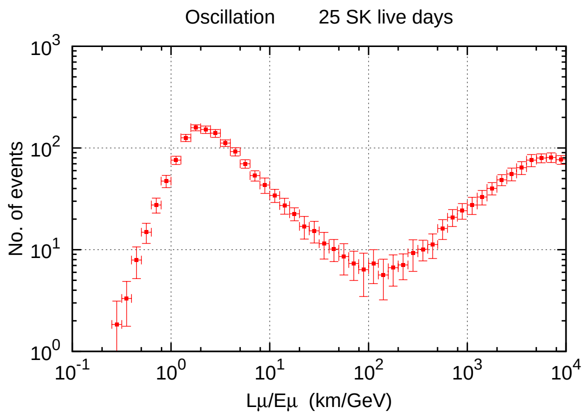

Fig. 16. L_μ/E_μ distribution with oscillation for 37230 live days (25 SK live days).

2.2.1 For null oscillation

In Figure 13, we give one sample for one SK live day (1489.2 live days) from the totally 37230 live days (25 SK live days) events, each of which has 1489.2 live days. Figure 14 shows the average distribution accompanied by the statistical uncertainty bar (not experimental error bar). It is clear from these figures that the existence of the dip or bottom, namely the sinusoidal character, means the contribution merely from horizontal contribution, having no relation with any neutrino oscillation character.

2.2.2 For oscillation (SK oscillation parameters)

In Figures 15 and 16, we give the L_μ/E_μ distributions with oscillation for 1489.2 live days (one SK live day) and 37230 live days (25 SK live days), respectively. In Figure 15, we may observe the uneven histogram, something like curious bottoms coming from neutrino oscillation. However, in Figure 16 where the statistics is 25 times as much as that of Figure 15, the histogram becomes smoother and such bottoms disappear, which turns out finally for the bottoms to be pseudo. It is impossible to extract the neutrino oscillation parameters from the comparison of Figure 16 with Figure 14.

In Figures 17 and 18, correspondingly, we give the correlation between L_μ and E_μ for 1489.2 live days (one SK live day) and 14892 live days (10 SK live days), respectively.

In Figure 19, we give the L_μ/E_μ distribution for 14892 live days (10 SK live days) in the linear scale which is another expression of the same content as in Figure 18. As in Figure 18, we cannot find any maximum oscillation-like phenomena in Figure 19, which is contrast to Figure 10.

It is clear from the figures that we can not observe the maximum oscillation in L_μ/E_μ, on the contrary to Figures 4 to 10 which give the maximum oscillations. Namely, we may conclude that we can not observe the sinusoidal flavor transition probability of neutrino oscillation against the claim by Super-Kamiokande Collaboration[4] when we adopt physically observable quantities, such as L_μ and E_μ.

In order to confirm the disappearance of the psuedo maximum oscillations, in Figures 20 and 21, we give the survival probability of a given flavor for L_μ/E_μ distribution, namely, $(L_\mu/E_\mu)_{osc}/(L_\mu/E_\mu)_{null}$, for 1489.2 live

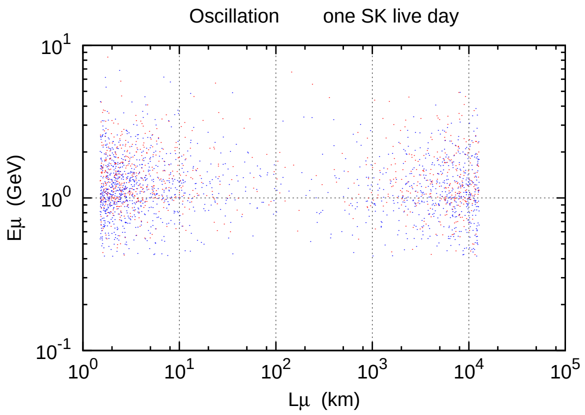

Fig. 17. The correlation diagram between L_μ and E_μ with oscillation for 1489.2 live days (one SK live day).

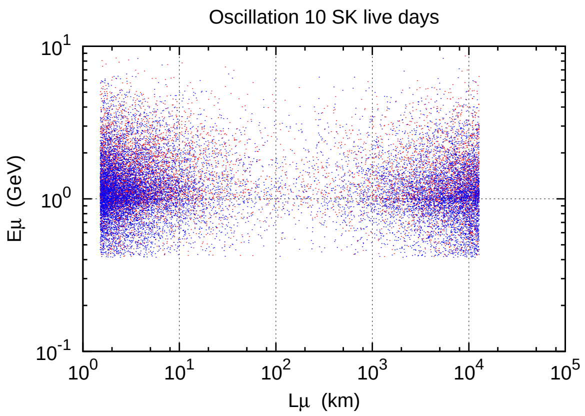

Fig. 18. The correlation diagram between L_μ and E_μ with oscillation for 14892 live days (10 SK live days).

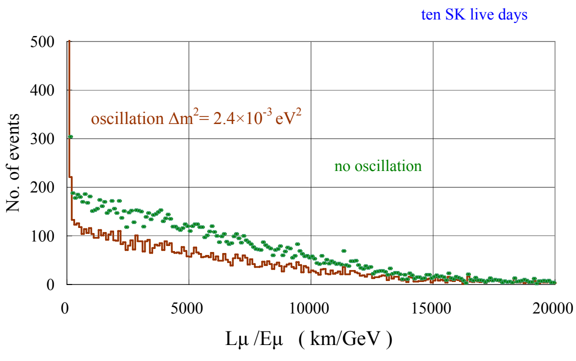

Fig. 19. L_μ/E_μ distribution with and without oscillation for 14892 live days (10 SK live days).

days (one SK live day) and 14892 live days (10 SK live days), respectively. In Figure 20, we show one example of $(L_\mu/E_\mu)_{osc}/(L_\mu/E_\mu)_{null}$ among 625 sets of ratios. Comparing Figure 20 with Figure 21, the pseudo dips in Figure 20 disappear in Figure 21. Thus the histogram becomes a rather decreasing function of L_μ/E_μ in Figure 21. If we further make statistics higher, the survival probabil-

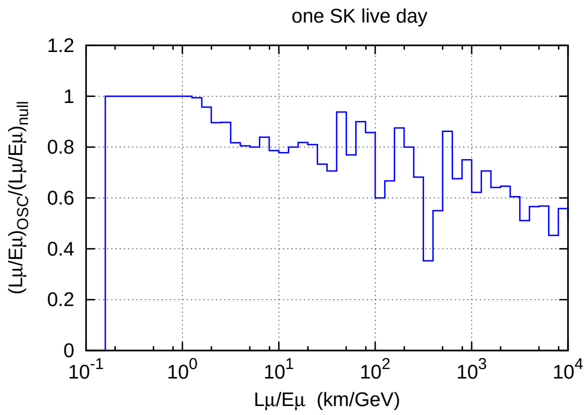

Fig. 20. The ratio of $(L_\mu/E_\mu)_{osc}/(L_\mu/E_\mu)_{null}$ for 1489.2 live days (one SK live day).

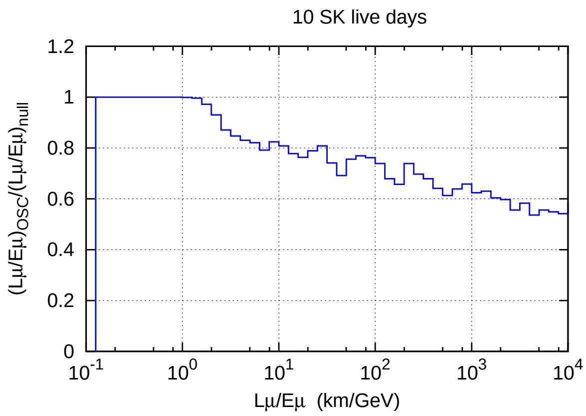

Fig. 21. The ratio of $(L_\mu/E_\mu)_{osc}/(L_\mu/E_\mu)_{null}$ for 14892 live days (10 SK live days).

ity for L_μ/E_μ distribution should be a monotonously decreasing function of L_μ/E_μ, whithout showing any characteristics of the maximum oscillation, which is in contrast to Figures 11 and 12.

In conclusion, we should say that we can not find any maximum oscillation for the neutrino oscillation in the L_μ/E_μ distribution.

2.3 L_μ/E_ν distribution

Now, we examine the L_μ/E_ν distribution which Super-Kamiokande Collaboration treat in the thier paper, expecting the evidence for the oscillatory signatuture in atmospheric neutrino oscillations.

2.3.1 For null oscillation

In Figures 22 and 23, we give the L_μ/E_ν distribution without oscillation for 1489.2 live days (one SK live day) and 37230 live days (25 SK live days), respectively. Comparing Figure 22 with Figure 23, the larger statistics makes the distribution smoother. Also, there is a sinusoidal-like bottom which has no relation with neutrino oscillation.

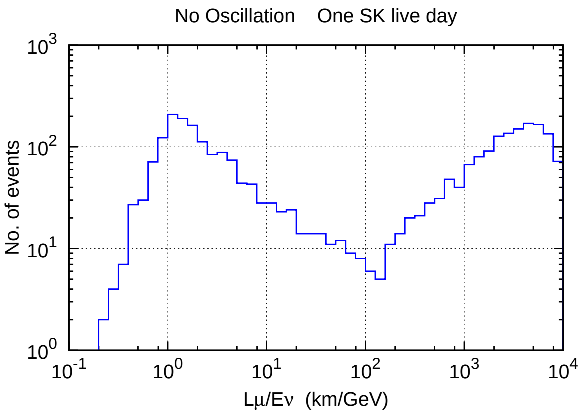

Fig. 22. L_μ/E_ν distribution without oscillation for 1489.2 live days (one SK live day).

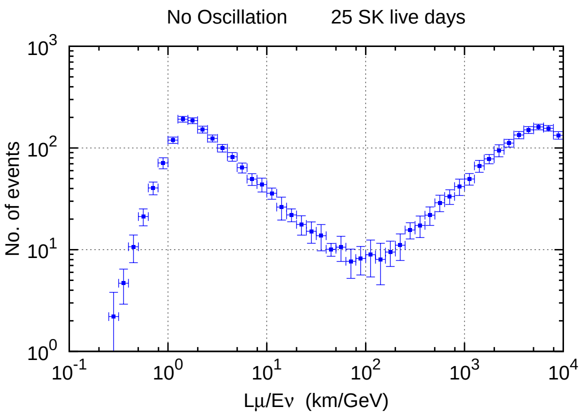

Fig. 23. L_μ/E_ν distribution without oscillation for 37230 live days (25 SK live days).

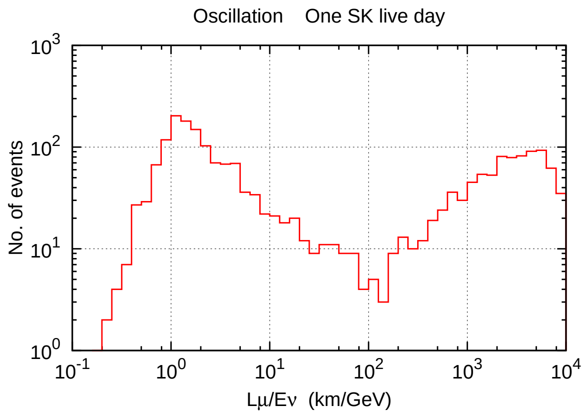

Fig. 24. L_μ/E_ν distribution with oscillation for 1489.2 live days (one SK live day).

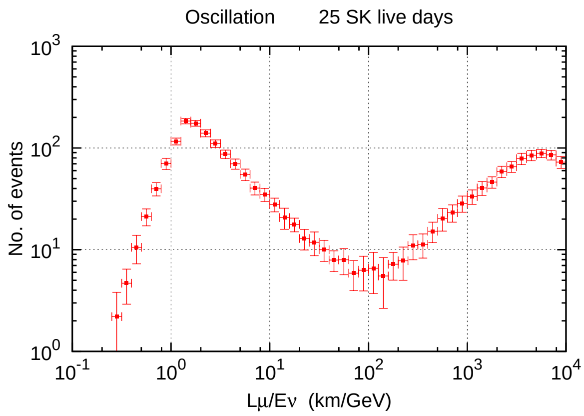

Fig. 25. L_μ/E_ν distribution with oscillation for 37230 live days (25 SK live days).

2.3.2 For oscillation (SK oscillation parameters)

In Figures 24 and 25, we give the L_μ/E_ν distribution with oscillation for 1489.2 live days (one SK live day) and 37230 live days (25 SK live days), respectively. In Figure 24, we may find something like a bottom which corresponds to the first maximum oscillation near $\sim$200 (km/GeV). However, such the dip disappears, by making the statistics larger as shown in Figure 25.

2.3.3 $L_\mu/E_{\nu,SK}$ distribution for the oscillation

Instead of E_ν which is correctly sampled from the corresponding probability functions, let us utilize $E_{\nu,SK}$ which is obtained from the "approximate" formula (Eq.(6) in the preceding paper[1]).

We express E_ν described in Eq.(6) of the preceding paper[1] utilized by Super-Kamiokande Collaboration as $E_{\nu,SK}$ to discriminate our E_ν obtained in the stochastic manner correctly.

In Figure 26, we give $L_\mu/E_{\nu,SK}$ distribution for 14892 live days (10 SK live days), comparing with L_μ/E_ν distribution. It is understood from the comparison that there

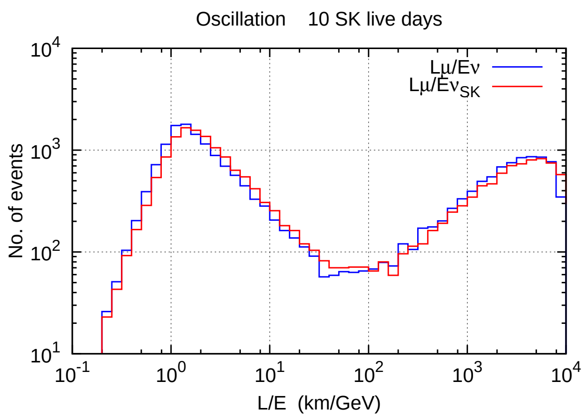

Fig. 26. The $L_\mu/E_{\nu,SK}$ distribution in comparison with L_μ/E_ν distribution with oscillation for 14892 day (10 SK live days).

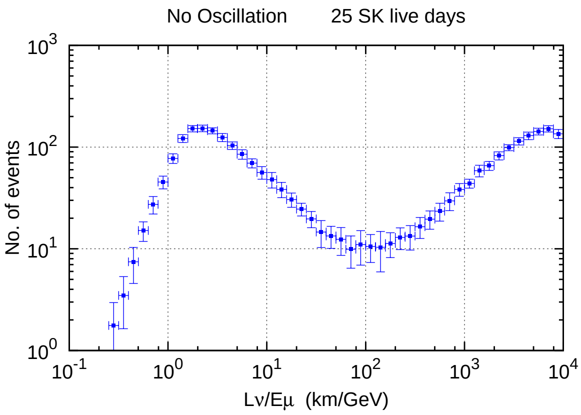

Fig. 27. The L_ν/E_μ distribution without oscillation for 37230 days (25 SK live days).

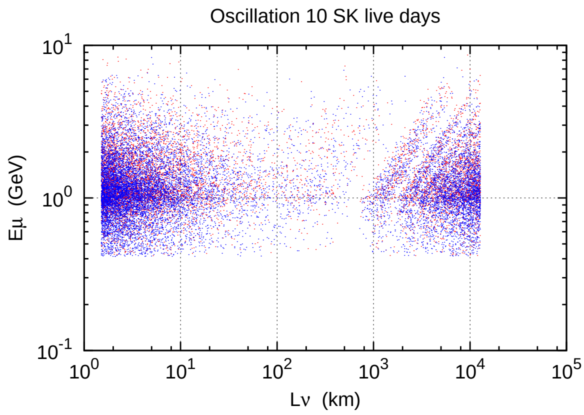

Fig. 29. The correlation diagram between L_ν and E_μ with oscillation for 14892 days (10 SK live days).

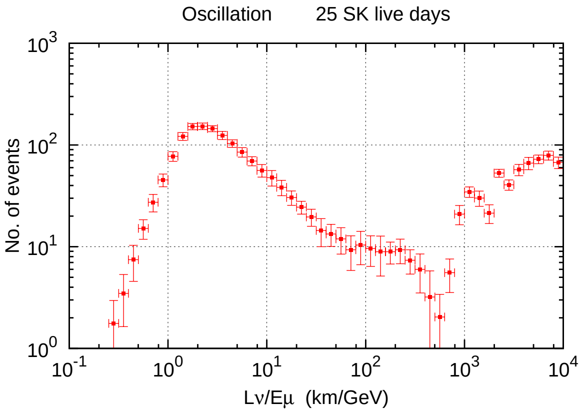

Fig. 28. The L_ν/E_μ distribution with oscillation for 37230 days (25 SK live days).

is no significant difference between $L_\mu/E_{\nu,SK}$ distribution and L_μ/E_ν one. This fact tells us that the "aproximate" formula for E_ν by Super-Kamiokande Collaboration does not produce so significant error practically. Although this kind of foumula is not suitable for the treatment of stochastic quantities, the result is understandable from Figure 14 in the preceding paper[1]. Also, we can conclude that we do not find any hole corresponding to the maximum oscillation in L_μ/E_ν or $L_\mu/E_{\nu,SK}$ distributions. The reason why the Figure 25 can not show such dip structure as shown in Figures 4 and 5, comes from the situation that the role of L_ν is much more crucial than that of E_ν in the L/E analysis. Namely, L_ν cannot be replaced by L_μ at all. Also, see the discussion in the following subsection 4.4.

2.4 L_ν/E_μ distribution

2.4.1 For null oscillation

In Figure 27, we give L_ν/E_μ distribution without oscillation for 37230 live days (25 SK live days) of Super-Kamiokande Experiment to consider statistical fluctuation effect as precisely as possible. It is clear from the figure that there is not any dip corresponding to the maximum oscillation which is expected to appear in presence of neutrino oscillation, as it must be.

2.4.2 For oscillation (SK oscillation parameters)

In Figure 28, we give the corresponding distribution with the oscillation. In Figure 29, we give the correlation diagram between L_ν and E_μ for 14823 live days (10 SK live days). On the contrary to Figure 25, there are surely some kinds of holes in Figure 28, and furthermore we can discriminate the strip pattern in Figure 29, similarly as in Figure 9.

Therefore, we surmise from Figures 28 and 29 that we may observe some "maximum oscillation like" quantities which are related to the maximum oscillations in the L_ν/E_ν distribution through the correlation between E_μ and E_ν shown in Figure 14 in the preceding paper[1]. However, it seems to be difficult to extract a pair of concrete values of L_ν and E_ν through the analysis of the L_ν/E_μ distribution. In Figure 30, we make a comparison between L_ν/E_ν distribution and L_ν/E_μ distribution where the correlation between E_ν and E_μ is shown in Figure 14 in the preceding paper[1]. It is clear from the figure that the L_ν/E_ν distribution demonstrates the maximum oscillation as already shown in Figures 4 to 10 and the L_ν/E_μ distribution also demonstrates the maximum oscillation-like as already shown in Figure 28 and 29. In Figure 31, we give the relation between L_μ/E_ν distribution and L_μ/E_μ distribution where the same correlation between E_ν and E_μ holds in the case of Figure 30. It is also clear from the figures that both the distributions demonstrate neither the maximum oscillation nor the maximum oscillation-like, which is also clear from Figures 15 to 19 and Figures 24 to 25. Thus, it can be concluded from Figures 13 and 14 in the preceding paper[1] and Figure 30 and Figure 31 in the present paper that L_ν plays an essential role compared with others L_μ, E_ν or E_μ. In other words, it should be noticed that L_ν cannot be approximated by L_μ, while E_ν can be obtained approximately from E_μ through some procedure. Also, such a serious discrepancy between

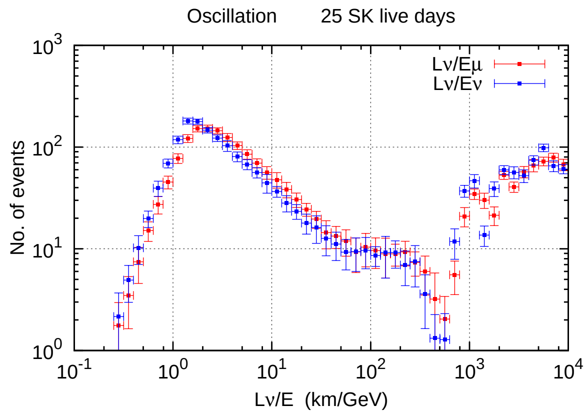

Fig. 30. Comparison between L_ν/E_ν distribution and L_ν/E_μ distribution with oscillation for 37230 days (25 SK live days).

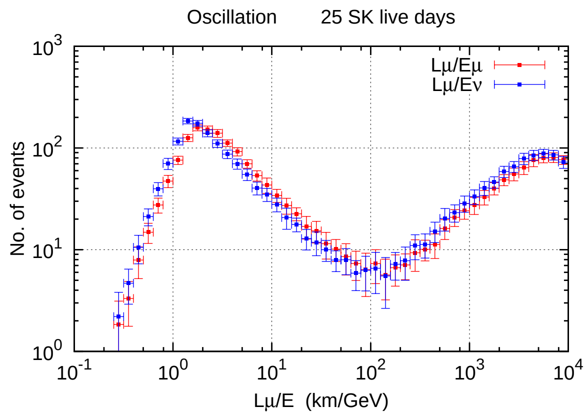

Fig. 31. Comparison between L_μ/E_ν distribution and L_μ/E_μ distribution with oscillation for 37230 days (25 SK live days).

L_ν-L_μ relation and E_ν-E_μ relation is shown in the comparison of Figure 30 with Figure 31.

3 Comparison of L/E Distribution in the Super-Kamiokande Experiment with our Results

In our classification, the L/E distribution by Super-Kamiokande Collaboration [2][4] should be compared directly with our L_μ/E_ν distributiton. Taking account of their assertion of existence of the maximum oscillation we compare their results with our results on L_ν/E_ν in Figure 32 [2]. It is clear from the figure that there are two big differences between them.

One is that we observe the first maximum oscillation ($L_\nu/E_\nu = 515$ km/GeV under the SK oscillation parameters) sharply, while SK observe it in the wider range of $L_\nu/E_\nu = 100 \sim 800$ km/GeV.

Such the lack of the neutrino events over the wide range may be due to their measurement of L_μ, but not

[2] We read out *Fully Contained Events* among total events from Super-Kamiokande Collaboration [2][4].

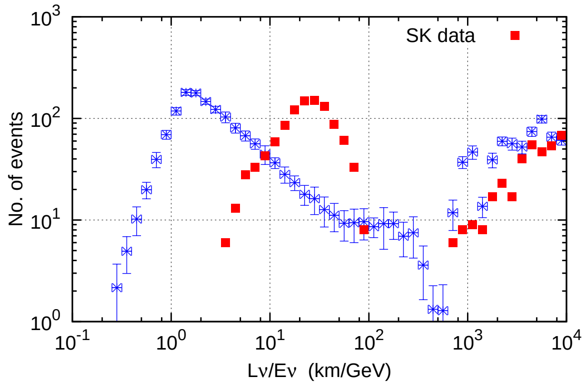

Fig. 32. The comparison of L/E distribution for single-ring muon events due to QEL among *Fully Contained Events* with the corresponding one by the Super-Kamiokande Experiment.

L_ν, because the given definite L_ν corresponds to L_μ over a wide range and vice versa (See also the correlation between L_ν and L_μ in Figure 34 and 37)

The other is that there is big difference between them as for the position which give the maximum frequency for the events concerned. Here, we do not mention to the existence of the maximum oscillation which is derived from the measurement of L_μ utilized in Super-Kamiokande Collaboration, because one cannot observe the maximum oscillation, if we utilize L_μ (see Figures 14 to 19). Conseqently, we examine the second point as for the maximum frequency for the events concerned. Our computer numerical experiment gives the maximum frequency for interval $1.0 < L_\nu/E_\nu < 1.26$ (km/GeV) as shown in Figure 32.

In Figure 33, we give the correlation between L_ν and E_ν for interval $1.0 < L_\nu/E_\nu < 1.26$ (km/GeV). It is clear from the figure that the larger part of the incident neutrino events is occupied by the vertically downward ones and the smaller part is occupied by the horizontally downward neutrino events. This is quite reasonable, because more intensive downward flux contribute to the maximum frequency for the events concerned, compared with weaker upward flux under the Super-Kamokande neutrino oscillation parameters.

In Figure 34, we give the correlation diagram between L_ν and L_μ for the same intervals as in Figure 33. It is clear from Figure 34 that the majority of the events is concentrated into the squared regions with $L_\nu < 10$ km and $L_\mu < 10$ km. This denotes that the downward incident neutrinos produce muons toward the forward direction with either smaller or larger angles and only the smaller part of the downward incident neutrino events produce the upward muons due to backscattering (1000 to 10,000 km in L_μ) as well as the azimuthal angle effect in QEL. In Figure 35, we give the correlation diagram between L_μ and E_μ for the same intervals as in Figure 33. It is clear from this figure that the produced muons with higher energies are ejected toward the forward and vertical-like directions, while the produced muon with lower energies may be ejected toward the backward or holizontal-like di-

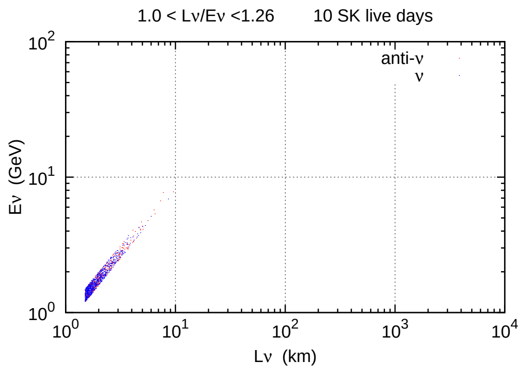

Fig. 33. Correlation diagram between L_ν and E_ν for $1.0 < L_\nu/E_\nu < 1.26$ (km/GeV) which corresponds to the maximum frequency of the neutrino events for L_ν/E_ν distribution in our computer numerical experiment for 14892 live days (10 SK live days).

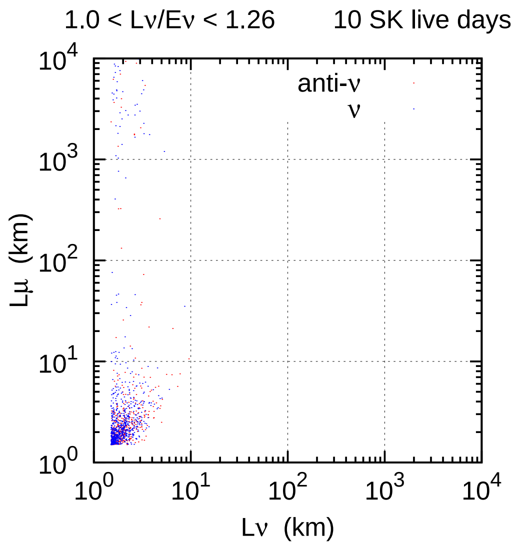

Fig. 34. Correlation diagram between L_ν and L_μ for $1.0 < L_\nu/E_\nu < 1.26$ (km/GeV) under the neutrino oscillation parameters obtained by Super-Kamiokande Collaboration for 14892 live days (10 SK live days).

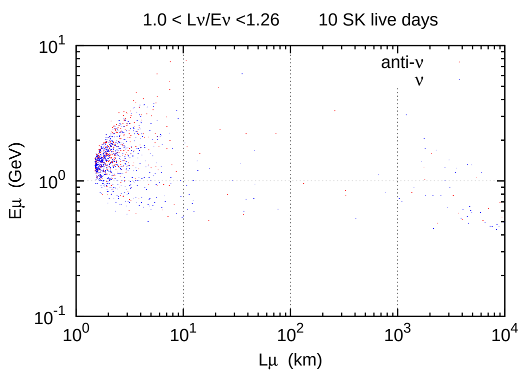

Fig. 35. Correlation diagram between L_μ and E_μ for $1.0 < L_\nu/E_\nu < 1.26$ (km/GeV) which corresponds to the maximum frequency of the neutrino events for L_ν/E_ν distribution in our computer numerical experiment for 14892 live days (10 SK live days).

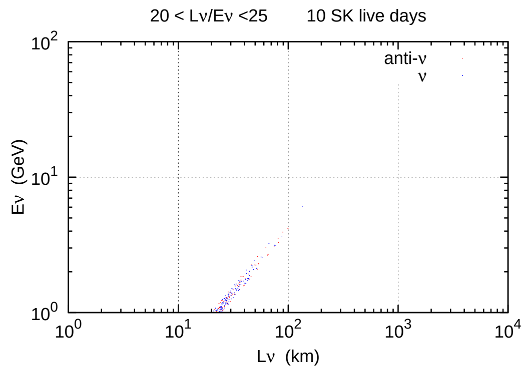

Fig. 36. Correlation diagram between L_ν and E_ν for $20 < L_\nu/E_\nu < 25$ (km/GeV) which corresponds to the maximum frequency of the neutrino events for L_μ/E_ν distribution in SK experiment for 14892 live days (10 SK live days).

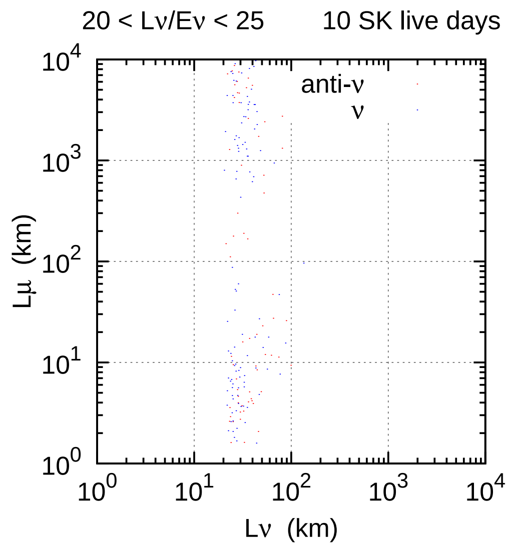

Fig. 37. Correlation diagram between L_ν and L_μ for $20 < L_\nu/E_\nu < 25$ (km/GeV) under the neutrino oscillation parameters obtained by Super-Kamiokande Collaboration for 14892 live days (10 SK live days).

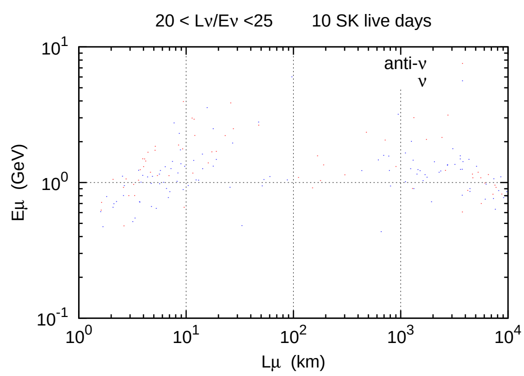

Fig. 38. Correlation diagram between L_μ and E_μ for $20 < L_\nu/E_\nu < 25$ (km/GeV) which corresponds to the maximum frequency of the neutrino events for L_ν/E_ν distribution in SK experiment for 14892 live days (10 SK live days).

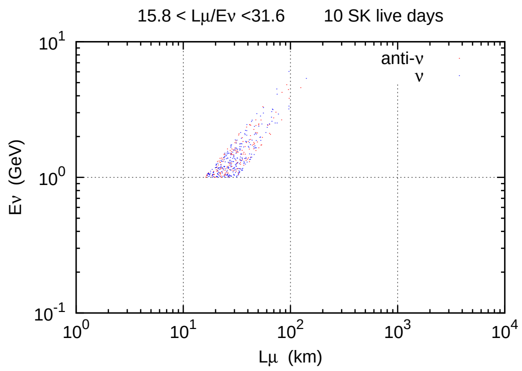

Fig. 39. Correlation diagram between L_μ and E_ν for $15.8 < L_\mu/E_\nu < 31.6$ (km/GeV) which correspond to the maximum frequency of the neutrino events for L_μ/E_ν distribution in SK experiment for 14892 live days (10 SK live days).

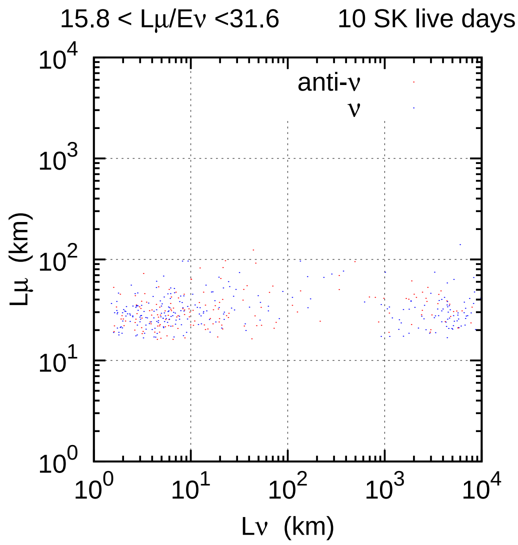

Fig. 40. Correlation diagram between L_ν and L_μ for $15.8 < L_\mu/E_\nu < 31.6$ (km/GeV) under the neutrino oscillation parmeters obtained by Super-Kamiokande Collaboration for 14892 live days (10 SK live days).

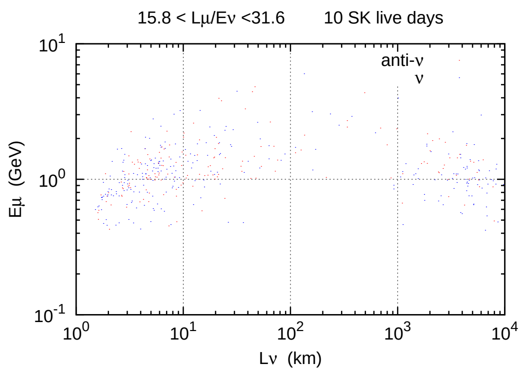

Fig. 41. Correlation diagram between L_ν and E_μ for $15.8 < L_\mu/E_\nu < 31.6$ (km/GeV) which correspond to the maximum frequency of the neutrino events for L_μ/E_ν distribution in SK experiment for 14892 live days (10 SK live days).

rection. Namely, it is concluded from Figures 33, 34 and 35 that the vertically downward neutrino events can contribute to the maximum frequency, because they are free from neutrino oscillation. It is further noted that the direction of the produced muon does not coincide with the original direction of the neutrino.

Now, we examine L-E relation at the position for our computer numerical expermiment where Super-Kamiokande Collaboration give the maximum frequency for the events ($20 < L_\nu/E_\nu < 25$ (km/GeV)). In Figure 36, we show the correlation between L_ν and E_ν for interval $20 < L_\nu/E_\nu < 25$ (km/GeV).

It is clear from Figure 36 that L_ν distribute over 27 $\sim$ 120 km, corresponding to $cos\theta_\nu$ = - 0.1 $\sim$ 0, which denotes the horizontal-like downward neutrino events. The frequency of the horizontal-like downward neutrino events in Figure 36 are pretty smaller than that of the vertical-like downward neutrino events in Figure 33 due to smaller solid angles. In Figure 37, we give the correlation diagram between L_ν and L_μ for $20 < L_\nu/E_\nu < 25$ (km/GeV). It is impressive from the figure that L_μ distribute over four orders of magnitude (2 km to 1.2×10^4 km), while L_ν cover within one order of magnitude (20 $\sim$ 120 km). This fact denotes that the effect of the azimuthal angles in QEL is pretty strong even in the horizontal-like downward neutrino events in which the produced muons are apparently judged to come from the upward direction (see Figure 3-c and Figures 8 to 10 in the preceding paper[1]).

In Figure 38, we give the correlation diagram between L_μ and E_μ for the same intervals as in Figure 36. If we compare Figure 36 with Figure 38, then we can find the following interesting situation. As it is clearly understandable from Figure 36, horizontal-like downward neutrinos produce the muons in the three different regions, namely, vertical-like downward muons, horizontal-like downward muons and upward muons. From horizontal-like downward neutrinos with rather low energies, the vertically downward muons are ejected with rather large scattering angles. On the other hand, the horizontal-like downward muons are ejected with rather small angles whose energies are close to the incident neutrinos energies. Furthermore, the upward muons are produced either due to backscattering or due to the azimuthal effect in QEL for horizontal-like incident neutrinos (see Figures 8 and 9 in the preceding paper[1]). Thus, from the comparison of Figures 33, 34 and 35 with Figures 36, 37 and 38, it is reasonable for the the maximun frequency of the L_ν/E_ν events to occur for $1.0 < L_\nu/E_\nu < 1.26$ (km/GeV), and not to occur for $20 < L_\nu/E_\nu < 25$ (km/GeV) where Super-Kamiokande Collaboration "assert".

Finally, we examine the correlation between L_μ and E_ν for $15.8 < L_\mu/E_\nu < 31.6$ (km/GeV) where Super-Kamiokande Collaboration give the maximum frequency of L/E neutrino events as shown in Figure 32. Although we compare their frequency with that of our L_ν/E_ν in Figure 32, we can compare their frequency with that of our L_μ/E_ν in Figure 31, which shows big difference between them. In Figure 39, we give the correlation diagram between L_μ and E_ν for $15.8 < L_\mu/E_\nu < 31.6$ (km/GeV). In

Figures 40 and 41, we give the corresponding correlation diagrams between L_ν and L_μ, and L_ν and E_μ, respectively.

It is clear from Figure 39 that Super-Kamiokande Collaboration measure the vertical-like downward muons. It is also clear from Figures 40 and 41 that these vertical-like downward muon are produced by the incident neutrinos whose L_ν are distributed over four orders of magnitude. These incident neutrinos are classified into two parts. One is the downward incident neutrinos ($1.0 < L_\nu < 100$ km) and the other ($L_\nu > 100$ km) is the upward incident neutrinos. The majority of the incident neutrino is occupied by the vertical-like downward. However, the frequency of the upward neutrinos is in the same order of the magnitude as the horizontal-like downward. The upward incident neutrinos may produce downward muons due to both backscattering and the azithumal angle effect in QEL. At any rate, for the measured muons in the case of the maximun frequency of the events, L_ν of the corresponding incident neutrinos distribute over four orders of magnitude. Shortly speaking, for the maximum frequency of the neutrino events $15.8 < L_\mu/E_\nu < 31.6$ (km/GeV), the magnitude of the L_μ of the produced muons lie within one order of magnitude (see Figure 39), although the L_ν of the incident neutrinos which produce these muons distribute over four orders of magnitude. In other words, it is concluded that Super-Kamiokande Collaboration do not measure the definite direction of the incident neutrinos as far as they measure L_μ. It is furthermore noticed from the comparison of Figure 41 with Figure 29 that Figure 41 is obtained from Figure 29 by cutting off the stripe of $15.8 < L_\mu/E_\nu < 31.6$ (km/GeV). Therefore ,we can recognize the vacant region of the neutrino events faintly in the part between 100 and 1000 (km/GeV) in Figure 41 which is clearly shown in Figure 29. The vacant region of the events shows indication of neutrino oscillation.

The summary on Figures from 33 to 41 are as follows; Figures from 33 to 35 represent the mutual relations among L_ν, L_μ, E_ν and E_μ near *our* maximum frequency of L_ν/E_ν distribution. Here, all the incident neutrinos are occupied by the downward vertical-like neutrinos, while the majority of the emitted muons is occupied by the downward muon and the minority is occupied by the upward muon. Figures from 36 to 38 represent the similar mutual relations for *our* L_ν/E_ν distribution which correspond to the near the maximum frequency of L_μ/E_ν distribution obtained by Super-Kamiokande Collaboration. Here, almost the incident neutrinos are occupied by the downward holizontal-like neutrinos, while about the half of emitted muons is recognized as the downward muon and the other half is done as the upward muon. Figures from 39 to 41 represent the mutual similar relations, assuming the numerical values of the maximum frequency of L_μ/E_ν distribution obtained by Super-Kamiokande Collaboration. Here, the majority of the emitted muons is occupied by the horizontal like muons, while their parent neutrinos come from both the downward neutrinos and the upward ones. The common characteristics through Figures from 33 to 41 is that for given definite $L_\nu(L_\mu)$ we find $L_\mu(L_\nu)$ which distribute over the four order of magnitudes.

4 Conclusion

The assumption made by Super-Kamiokande Collaboration that the direction of the reconstructed lepton approximately represents the direction of the original neutrino does not hold even approximately [5]. This is logically equivalent to the statement that L_ν cannot be replaced by L_μ even if approximately. This is really clarified in Figures 12 and 13 in the preceding paper[1].

Although the derivation of E_ν from E_μ (Eq.(6) of the preceding paper[1]) is theoretically, irrelevant to the stochastic plobrem, because of the neglect of the stochastic character in physical processes concerned, such the approximation does not induce so practically serious error compared with the assumption of $L_\nu \approx L_\mu$. As clarified in Figures 4 to 12, the maximum oscillation in L/E analysis can be observed only in the L_ν/E_ν distribution and it is quite natural by the definition of the probability for a given favor whose argument is L_ν/E_ν (Eq.(1)). As clarified in Figures 15 to 19 and Figures 24 to 26 the maximum oscillation for the presence of neutrino oscillation cannot be observed from both L_μ/E_μ and L_μ/E_ν. The relation between L_ν and L_μ is too complicate to extract similar expression to Eq.(6) of the preceding paper[1] for the argument on L_μ/E_μ and L_μ/E_ν . Similarly in the case of argument of L_ν/E_ν, we can indicate something like the maximum oscillation in L_ν/E_μ distribution which are shown in Figures 2741 and 28. The situation is derived from the fact that what plays a decisive role in L/E analysis is L_ν, but not E_ν, which are clearly shown by comparing Figures 12 and 13 with Figure 14 in the preceding paper[1].

As for L/E distribution obtained by Super-Kamiokande Collaboration, we definitely indicate that the maximum oscillation cannot be observed through the measurement of L_μ. Consequently, we cannot observe the maximum oscillation in L/E analysis which is carried out in Super-Kamiokande Collaboration. Furthermore, one cannot find the maximum frequency of L/E events at the position where Super-Kamiokande Collaboration observe, even if one can observe L_ν.

In conclusion, the maximum oscillation in L/E analysis can be observed only in L_ν/E_ν, but not in any other combinations of L with E. However, L_ν is physically unobservable quantities and it cannot be approximated by L_μ, because the assumption between L_ν and L_μ, does not hold even if statistically. Consequently, it should be concluded that Super-Kamiokande cannot observe the maximum oscillation in their $L_\mu/E_{\nu,SK}$ analysis.

Finally, our conclusion that L_ν cannot be approximate by L_μ is logically equivalent to the statement that $cos\theta_\nu$ cannot be approximated by $cos\theta_\mu$, where $cos\theta_\nu$ denotes cosine of the zenith angle of the incident neutrino and $cos\theta_\mu$ denotes that of the produced muon, respectively [5]. In Super-Kamiokande Collaboration, they approximate $cos\theta_\nu$ as $cos\theta_\mu$ (See the reproduction of their statements in the 2 page in the present paper). The analysis of the zenith angle distribution of the atmospheric neutrino events by Super-Kamiokande Collaboration will be re-examined in our subsequent papers.

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